Heterogeneity in synchronizing networks of mobile particles

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Introduction: Fujiwara model

We consider a system of particles moving on a 2D square of size *L*x*L*. Particles will interact with any particle located at a distance less than *d* (Fig. 1A). The dynamics is determined by two ingredients:

■ Phase synchronization in the spatial network:

$$\phi_i(t+1) = \phi_i(t) + \sigma \sum_{j, d_{ij} < d} \sin[\phi_j(t) - \phi_i(t)]$$

where $\sigma = 0.005$ is the coupling strength, and d_{ij} is the Euclidean distance between particles *i* and *j*.

■ Mobility:

$$x_i(t_k + \Delta t) = x_i(t_k) + v\cos(\xi_i(t_k))\Delta t/\tau_M \mod L$$
$$y_i(t_k + \Delta t) = y_i(t_k) + v\sin(\xi_i(t_k))\Delta t/\tau_M \mod L$$

Particles change randomly and periodically their motion direction.

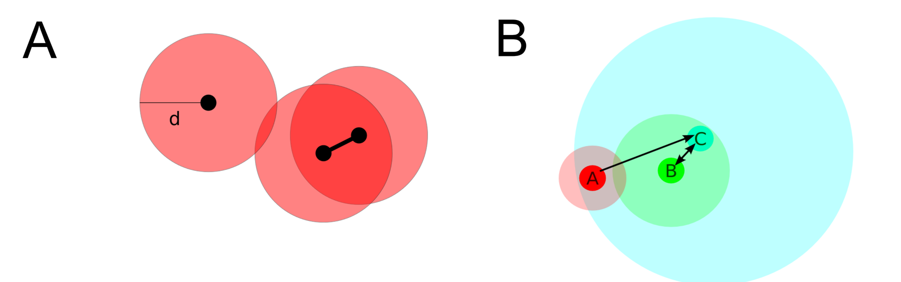

Figure 1. A) Interactions between nodes separated by distance less than *d*. B) Each node has a particular interaction distance, so interactions are non-symmetric (B and C interact with each other, A influences C, but C does not influence A).

Discrete heterogeneous velocities

With probability *p* [1-*p*], particles move with velocity v_0=10 [v_1=100]. We characterize the dynamics with the average phase difference $\langle\Delta\phi\rangle$. The synchronization in the case of heterogeneous velocities (*p*≠0,1) is slower than for homogeneous velocities with $\overline{v} = pv_0 + (1-p)v_1$ (Fig. 3). There is a characteristic synchronization time n_T ($\langle\Delta\phi\rangle \sim e^{-t/n_T}$). Synchronization is faster for higher values of *p*, and for higher values of *d* (Fig. 4).

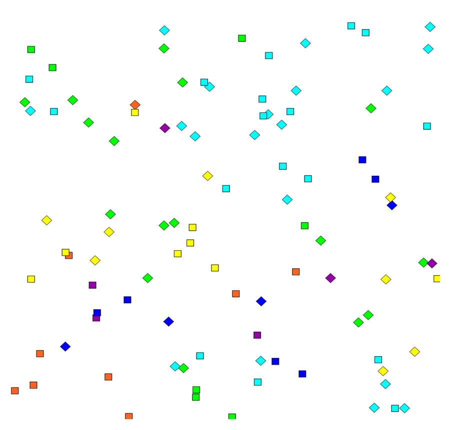

Figure 2. QR code with a link to a movie (http://youtu.be/5kynQMk-IgQ), and snapshot at t=200. Diamonds and squares are, respectively, fast and slow particles (*p*=0.5), and color indicates phases. More information in http://ifisc.uib-csic.es/users/jorge/mobilepart.html

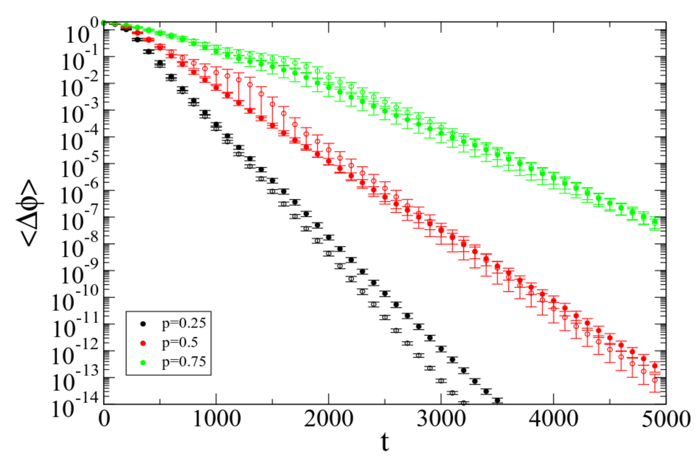

Figure 3. Average phase difference evolution for different values of p. Empty and filled symbols correspond, respectively, to homogeneous and heterogeneous systems, L=200, N=100.

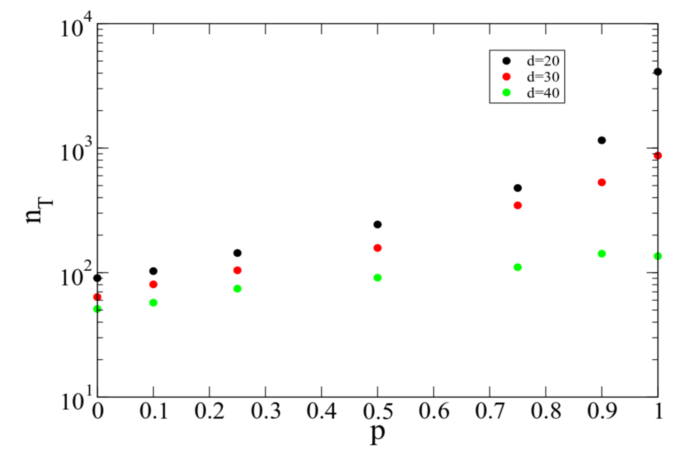

Figure 4. Characteristic synchronization time as a function of p for different values of the interaction radius, L=200, N=100.

Continuous heterogeneous velocities

We consider a Gaussian distribution of velocities centred in *v*=40 of width γ, and we study the change in the synchronization time as the width is varied. Figure 4 shows how 1/n_T increases with γ, i.e. the process is faster as heterogeneity is continuously increased ($n_T^{-1} = A + B\gamma + C\gamma^2$). Increasing the heterogeneity in the velocities, we decrease the time duration of the links. The relation between n_T and the average link time $\langle\tau\rangle$ is linear (as the interactions last more, the global synchronization process is slower (Fig. 5)). The average link time duration is inversely proportional to the average relative velocity $\langle||v_1 - v_2||\rangle$, and this quantity behaves parabolically with the width of the velocity distribution (Fig. 6). Summing up, the parabolic behaviour of 1/n_T with the width comes from modifying the average relative velocity and, hence, the average time that an interaction lasts. A similar analysis has been performed in the system with discrete heterogeneous velocities, obtaining the same conclusion: changing the velocity features modifies the average link time duration and, hence, influences the characteristic synchronization time.

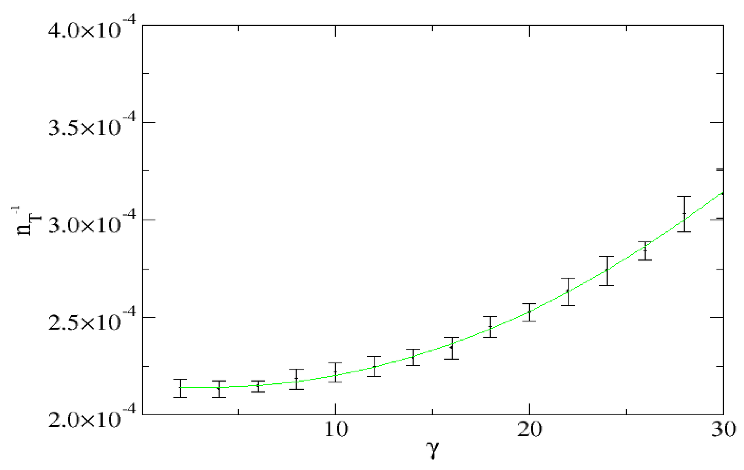

Figure 4. Inverse of the synchronization time as a function of the width of the velocity distribution. The green line represents a parabolic fit, L=800, N=1600, d=20.

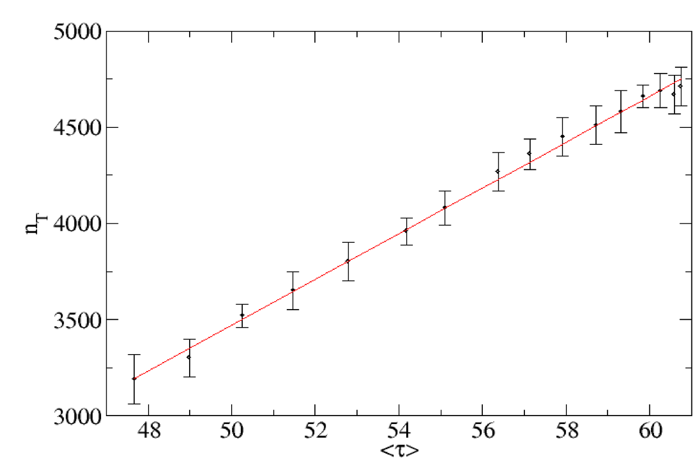

Figure 5. Synchronization time as a function of the average link time duration. The red line represents a linear fit, L=800, N=1600, d=20.

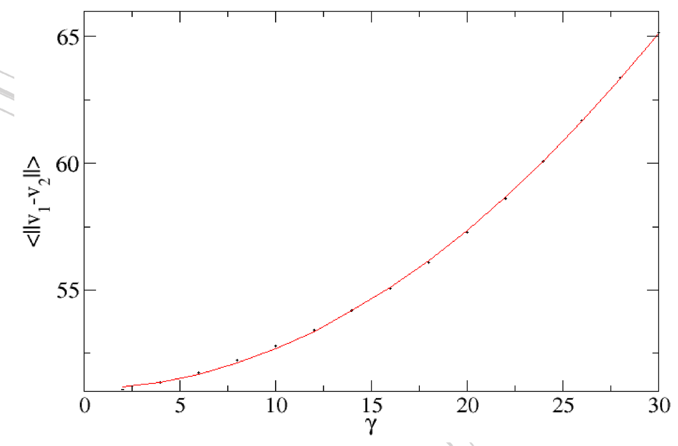

Figure 6. Numerical calculation of the average relative velocity between any two nodes as a function of the width of the velocity distribution. The red line represents a parabolic fit.

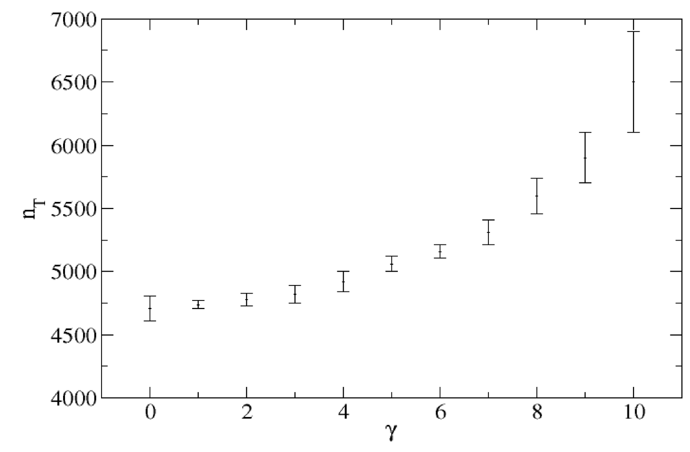

Figure 7. Synchronization time for a system with heterogeneous velocities and radii of interaction, as a function of the width of the distribution of velocities, L=800, N=1600, d_0=20.

Finally, we study a special case: faster nodes have a larger radius of interaction ($v_i = v_2 + \Delta v_i, d_i = d_0 + \frac{d_0}{v_2}\Delta v_i$). This set up makes the interactions be non-symmetric (node *i* might be influenced by *j* without *j* feeling the presence of *i*, Fig. 1B). We find that, as we increase the width of the Gaussian distribution, the synchronization process is slower, and eventually n_T diverges (no global synchronization, Fig. 7).

Conclusions

■ A system of particles with discrete heterogeneous velocities synchronizes slightly slower than one with a homogeneous average velocity.
■ If the distribution of velocities is continuous, the synchronization time decreases with the width of the distribution.
■ Modifying the velocity distribution changes the average relative velocity between any two nodes, influencing the average time that the interactions last. **If the nodes change their interacting neighbours more frequently, global synchronization is faster.**
■ Setting up a system in which faster nodes have larger zones of interaction, there is no global synchronization for high levels of heterogeneity.

References

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